Product Design Theory

& Basic Engineering Properties

CRANE MATERIALS INTERNATIONAL

A Whitepaper by:

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CMI provides product design innovations through extensive research and development. Our engineering personnel scrutinize every product extensively to ensure that each product meets and exceeds the performance standards our customer's desire. It is of paramount importance that all factors of the products performance are analyzed carefully. Our products are not only designed with a simplified theoretical overall bending strength, but all aspects of cross sectional loading and buckling, as well as ancillary stresses and deformations. CMI products are analyzed and tested by advanced computer modeling and Finite Element Analysis. Finished product performance is proven through extensive internal and third party testing of both material coupon cut outs, and full length full product sections. While testing and analyses are important factors in ensuring the performance and stability of CMI products, a customer should feel secure in purchasing a CMI product; even when these tests and analyses become confusing or monotonous. This paper provides for the customer the fundamentals basis for CMI product design.

Moment of Inertia and Bending Moment

The bending performance of a particular beam is largely controlled by a cross section property known as the second moment of area or more commonly known as the moment of inertia, *I*. The moment of inertia is based solely on the shape of a cross-section, or area, and not controlled whatsoever by material properties. The units for moment of inertia are most commonly measured in inches to the power of four (in^4). In the following calculation, the moment of inertia of area *A* is calculated about axis x.

$$I_x = \int_A y^2 dA$$

When a component is subjected to beam loading (i.e. the loading is applied perpendicular to the components longitudinal axis), there is an occurrence known as a bending moment induced in the beam. The induced bending moment is a factor of the supporting and loading conditions only, and not material properties or beam cross-section, and varies through the length of the beam.

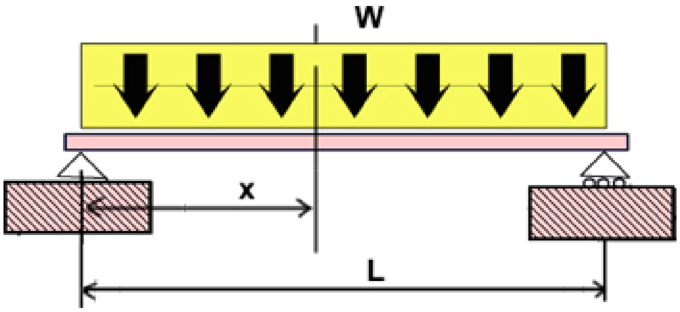

Figure 1
Simply Supported Beam with Uniformly Distributed Load

For a simply supported beam with a uniformly distributed load (*Figure 1*), the maximum bending moment occurs at mid span. Bending moment can be visualized as a force applied at a distance or moment arm and is usually reported in foot-pounds (ft-lbs). M_{max} is the maximum moment, *w* is the distributed load, and *L* is the span.

$$M_{\max} = \frac{wL^2}{8}$$

The loading configuration of a sheet piling wall can be complicated; therefore for this section we will examine more simplified loading cases for illustration purposes. Please refer to other CMI white papers for more detailed information on sheet piling loading configurations.

Stress and Strain

The structural performance of all materials is primarily controlled by two main factors:

Stress (σ) – Applied force over a given area, usually given in pounds per square inch (psi)

Strain (ε) – Amount of deformation or stretch of a material, usually given in inch per inch (in/in) or percentage (%)

Hooke's law relates stress and strain for materials that have identical values of a property in all directions, also known as isotropic materials. Hooke's law states that stress is proportional to strain. The slope of the stress-strain curve (*Figure 2*) is the elastic modulus or modulus of elasticity, *E*, and is given in pounds per square inch. *E* is a number that describes the amount of stress required for a unit elongation, or more simply put, the material stiffness. The elastic modulus is also known as Young's modulus and only applies up to the proportional limit of the material. The proportional limit is the maximum stress for which stress is proportional to strain, shown as point *P*. The elastic modulus, *E*, is a material property only and is not controlled by cross sectional, or other shape, parameters.

$$E = \frac{Stress}{Strain} = \frac{\sigma}{\varepsilon} \quad \text{or} \quad \sigma = E\varepsilon$$

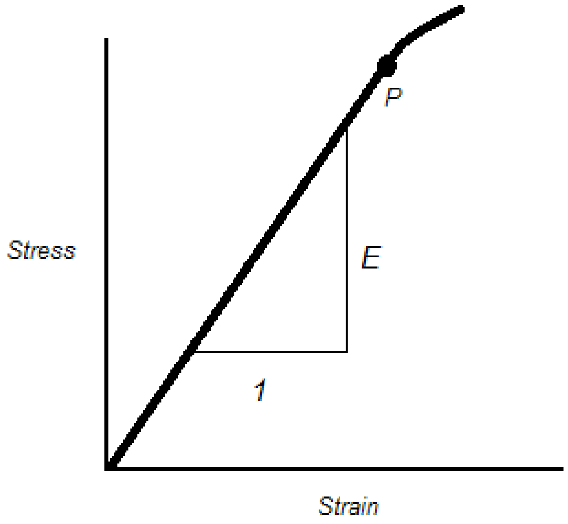

Figure 2
Stress-Strain Curve following Hooke's Law

With the exception of brittle materials, high values of *E* typically correspond to stiffer materials. Low values of *E* are acknowledged with more flexible materials.

Bending Stress

The bending stresses induced in the cross section of the beam are primarily tensile stresses in the bottom section reaching a maximum at the bottom edge, and compressive stresses in the top section reaching a maximum at the top edge (*Figure 3*). For a symmetric cross section, *c* can be assumed to be half of the cross section depth.

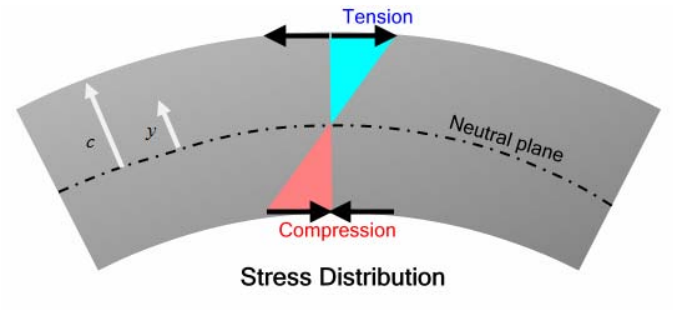

Figure 3
Bending Stress Distributions

The bending stress formula, also known as the flexure formula, gives the maximum normal stress on a cross section that has a bending moment *M*. The bending stresses can be calculated as shown, where *y*, the distance of the stress element from the neutral axis of the cross section. This formula gives the normal stress at any point on a cross section.

$$\sigma = \frac{My}{I}$$

Since we are generally interested in the maximum induced stress, or the case where $y_{max} = c$, the formula can be represented as

$$\sigma_{\max} = \frac{Mc}{I}$$

Both *I* and *c* are constants for a given cross section. In order to simplify calculations, a new constant, *Z*, can be substituted into the bending stress formula for symmetric cross sections. This new constant is called the elastic section modulus. Section modulus is usually reported in inches cubed (in^3), and is a shape property only.

$$\sigma_{\max} = \frac{M}{Z}$$

The stress-strain curves shown in *Figure 4* are for three different kinds of material: Brittle, Ductile, and Low-Carbon Steel. There are several points of interest that can be identified on these curves:

Proportional Limit, P: The maximum stress for which stress is proportional to strain, or the point at which the stress-strain curve becomes non-linear.

Yield Point, Y: The stress for which the strain increases without an increase in stress, or the point at which a material begins to plastically deform. All deformation below this point is recoverable, and the material will relax into its initial shape when the load is removed. For stresses above the yield point, a portion of the deformation is not recoverable, and the material will not relax into its initial shape. This unrecoverable deformation is known as plastic deformation.

Yield Strength, YS: The stress that will cause the material to undergo a certain specified amount of permanent deformation after unloading.

Ultimate Strength, U: The maximum amount of stress a material can support up to failure. After a ductile material has been loaded to its ultimate strength it begins to "neck" as the cross-sectional area of the specimen decreases due to plastic flow. Necking is accompanied by a region of decreasing stress with increasing strain on the stress-strain curve.

Breaking Strength, B: The stress in the material based on original cross-sectional area at the time it breaks. Also known as the fracture or rupture strength.

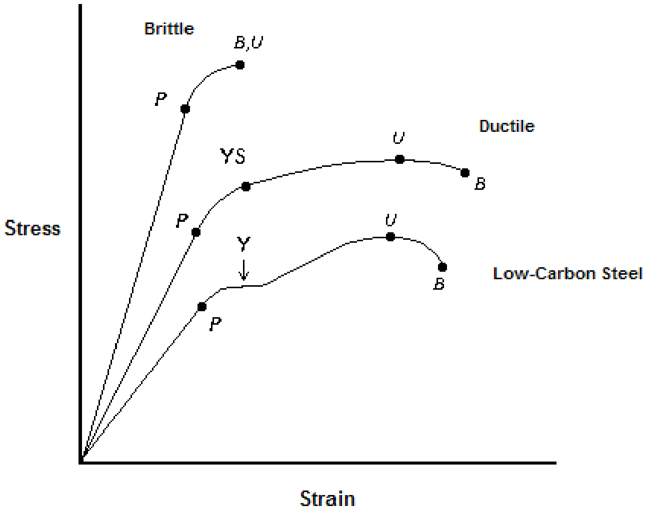

Figure 4
Stress-Strain Curves for Different Materials

Ductile metals other than Low-Carbon Steel typically do not have well-defined yield points. Brittle materials do not have yield points, and do not strain-harden, which means that the ultimate strength and breaking strength are the same.

Deflection

The deflection of a beam is principally a function of the moment of inertia of the beam cross-section, and the modulus of elasticity of the beam material. Generally speaking, the higher the moment of inertia and modulus of elasticity of a particular beam, the lower the deflection and therefore stiffer the beam will be in bending.

For the situation noted in *Figure 1*, the maximum deflection will occur at the center of the span and can be calculated in the following equation. Δ_{max} is the maximum deflection usually reported in inches (in). For deflection information on CMI products, refer to other CMI white papers.

$$\Delta_{\max} = \frac{5wL^4}{384EI}$$

Definition of Terms

Bending Moment, M - The primary term that describes bending forces in a beam.

Breaking Strength, B - The stress in the material based on original cross-sectional area at the time it breaks. Also known as the fracture or rupture strength.

Brittle - A materials is brittle if it is liable to fracture when subjected to stress.

Compressive Stress, σ_c - The stress applied to materials resulting in their compaction (decrease of volume).

Deflection - The degree to which a construction or structural element bends under a load.

Ductile - A material is ductile if it is capable of sustaining large plastic deformations without fracture.

Modulus of Elasticity, E *(Also known as "Young's" Modulus)* - The mathematical description of an object or substance's tendency to be deformed when a force is applied to it, or more simple put the stiffness or resistance of a material to loads. The elastic modulus of an object is defined as the slope of its stress-strain curve.

Moment of Inertia, I - The primary number which describes the bending performance of a particular shape.

***Proportional Limit, P* -** The maximum stress for which stress is proportional to strain, or the point at which the stress-strain curve becomes non-linear.

Section Modulus, Z - The primary shape property used to describe capacity of a cross section

Strain, ε - The amount of deformation or stretch of a material. Usually given in inch per inch (in/in) or percentage (%)

Stress, σ - Applied force over a given area. Usually given in pounds per square inch (psi)

Tensile Stress, σ_t - The stress state leading to expansion; that is, the length of a material tends to increase in the tensile direction.

Ultimate Strength, U - The maximum amount of stress a material can support up to failure.

Yield Point, Y - The stress for which the strain increases without an increase in stress, or the point at which a material begins to plastically deform.

Yield Strength, YS - The stress that will cause the material to undergo a certain specified amount of permanent deformation after unloading.